

# Using Item Scores and Response Times in Person-Fit Assessment

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# Introduction

- Person-fit assessment is used to identify individuals displaying unusual response behavior
- Several person-fit statistics have been developed for item scores, but few have been developed for item RTs and even fewer have been developed for item scores and RTs

**Table 1.** Existing Person-Fit Statistics.

Approach	Data Source		
	Item Scores	Item RTs	Item Scores & RTs
Frequentist	$I_s^*$	$I_t^*$	—
Bayesian	$p_s$	$p_t$	$p_{st}$

Hierarchical framework (van der Linden, 2007)

- 2PL model for the item scores
- Lognormal model for the item RTs
- A bivariate normal distribution for the person parameters, ability ( $\theta$ ) and speed ( $\tau$ )

## Purpose

Develop two frequentist methods for assessing person-fit in item scores and RTs.

- ① Combining individual person-fit statistics
- ② Joint model person-fit statistic

# Method

## Combining Individual Person-Fit Statistics

### Objective

Compute two individual person-fit statistics (one for the item scores, and one for the item RTs), and then combine them to form a single statistic.

- Item scores:  $I_s^*$  (Snijders, 2001)
- Item RTs:  $I_t^*$  (Sinharay, 2018)

# Method

## Combining Individual Person-Fit Statistics

- Problem:  $I_s^*$  and  $I_t^*$  exist on two different metrics
  - $I_s^*$  has an asymptotic  $\mathcal{N}(0, 1)$  null distribution
  - $I_t^*$  has a  $\chi_{n-1}^2$  null distribution
- Transform using the inverse CDF method
  - $q_s^*$  has an asymptotic  $\chi_1^2$  null distribution
  - $q_t^*$  has a  $\chi_1^2$  null distribution
- Their sum has an asymptotic  $\chi_2^2$  null distribution

$$q_{st}^* = q_s^* + q_t^* \quad (1)$$

# Method

## Joint Model Person-Fit Statistic

### Objective

Compute a single person-fit statistic using the likelihood function of the joint model for item scores and RTs.

- Standardized log-likelihood statistic (to be used with  $\theta$  and  $\tau$ )

$$I_{st} = \frac{I - E[I]}{\sqrt{\text{Var}(I)}} = \frac{W_n}{\sqrt{n}\sigma_n} \quad (2)$$

- Asymptotically correct version (to be used with  $\hat{\theta}$  and  $\hat{\tau}$ )

$$I_{st}^* = \frac{W_n + c_n s_0}{\sqrt{n}\tilde{\sigma}_n} \quad (3)$$

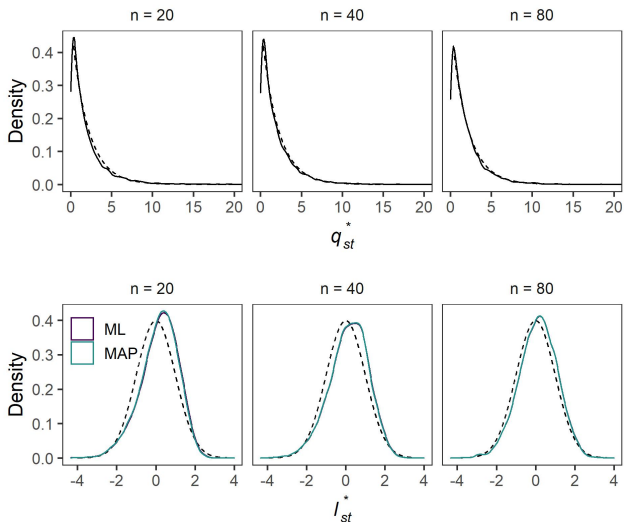


# Simulation Studies

- Study 1: The Null Distributions of  $q_{st}^*$  and  $l_{st}^*$
- Study 2: Performance of the Person-Fit Statistics

# Simulation Studies

## Study 1: The Null Distributions of $q_{st}^*$ and $l_{st}^*$



# Simulation Studies

## Study 2: Performance of the Person-Fit Statistics

- 1,000 examinees
  - 90% non-aberrant
  - 10% aberrant
- 100 replications
- LNIRT package in R
- Test length
  - 20
  - 40
  - 80
- Percentage of contaminated items
  - 10
  - 20
  - 40
- Correlation between  $\theta$  and  $\tau$ 
  - 0.2
  - 0.5
  - 0.8

# Simulation Studies

## Study 2: Performance of the Person-Fit Statistics

- Type I error rates decreased and power increased as...
  - test length increased
  - the percentage of contaminated items increased
- Across all conditions,  $q_{st}^*$  and  $I_{st}^*$  displayed satisfactory Type I error rates and larger power than the existing person-fit statistics

# Simulation Studies

## Study 2: Performance of the Person-Fit Statistics

**Table 2.** Power (40-Item Test,  $\alpha = 0.05$ ).

	Existing		New	
	$I_s^*$	$I_t^*$	$q_{st}^*$	$I_{st}^*$
Aberrance				
Preknowledge	.176	.309	.344	.350
Random responding	.314	.882	.896	.899

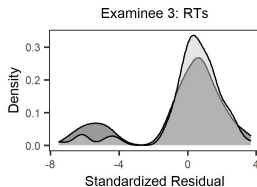
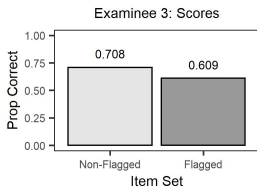
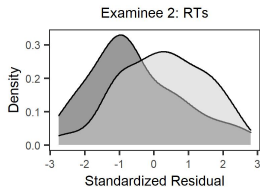
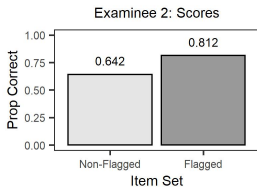
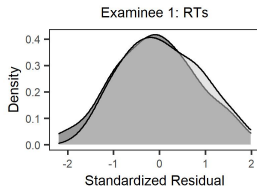
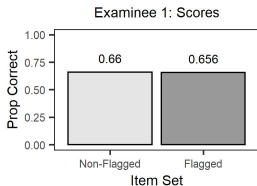
# Real Data Example

- Form 1 of the credentialing data set of Cizek and Wollack (2017)
- 1,624 examinees (41 flagged), 170 items (64 flagged)

**Table 3.** Proportions of Statistically Significant Values ( $\alpha = .05$ ).

Examinee Group	$q_{st}^*$	$I_{st}^*$
Non-Flagged	.196	.184
Flagged	.317	.268

# Real Data Example



# Conclusion

- We developed two frequentist person-fit statistics for item scores and RTs
- Appear to be promising tools for detecting aberrant behavior
- Future directions
  - Additional simulation conditions and real data sets
  - Investigate differences between  $q_{st}^*$  and  $l_{st}^*$
  - Extensions that utilize additional process data



# References

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